Addition Angular Momenta

Problem 1

Angular momenta l_1 and l_2 of two weekly interacting systems are combined into a resulting angular momentum with the value L. Show that in such states (with a definite L) the products $\hat{\mathbf{l}}_1 \cdot \hat{\mathbf{l}}_2$, $\hat{\mathbf{l}}_1 \cdot \hat{\mathbf{L}}$, $\hat{\mathbf{l}}_2 \cdot \hat{\mathbf{L}}$ have definite values as well.

Problem 2

Two weakly interacting systems have states characterized by quantum numbers $(l_1,\ m_1)$ and $(l_2,\ m_2)$ of their angular momenta and its z-projection. Give possible values, L, of the total angular momentum of a composite system $(1+2)$ and calculate the mean values $\bar{\mathbf{L}}$ and $\overline{\mathbf{L}^2}$. For the specific case where $m_1 = l_1,\ m_2 = l_2 - 1$ find the probabilities for the different possible values of the total angular momentum.

Problem 3

The problem of the addition of angular momenta of two systems l_1 and l_2 into the total angular momentum L could be solved by the following relation

$$\psi_{LM} = \sum C^{LM}_{l_1 m_1 l_2 m_2} \psi^{(1)}_{l_1 m_1} \psi^{(2)}_{l_2 m_2},\ M = m_1 + m_2$$

where $C^{LM}_{l_1 m_1 l_2 m_2}$ are the Clebsch–Gordan coefficients.

Using the "raising" ("lowering")-operators, $\hat{L}_{\pm}$, determine these coefficients for the special case of $L = l_1 + l_2$.

Problem 4

For a system of two angular momenta, $l_1 = l_2 = 1$, find the wavefunction, $\psi_{L=0}$ with $L = 0$ total angular momentum, using the projection operators.