Test 1

Systems of linear equations

Problem 1.1 Solve the system: $\begin{cases} 4x-4y+3z=-3 \\ 2x+3y-z=-9 \\ 6x-y+3z=-13 \end{cases}$

and find the sum: $x+y+z.$

Problem 1.2 Find the general solution of the system: $\begin{cases} 5x-y+3z=-3 \\ x+3y-3z=3 \\ 11x+y+3z=-3 \end{cases}$

Problem 1.3 Find the general solution of the system: $\begin{cases} 4x_1+3x_2-x_3+3x_4=-2 \\ 2x_1-3x_2+5x_3-3x_4=4 \\ 10x_1+3x_2+3x_3+3x_4=0 \end{cases}$

Problem 1.4 Given the system: $\begin{cases} 3x-y+3z=-1 \\ 6x+(6+a)z=2 \\ 3x+y+(7+a)z=-5 \\ 4y+(3a+1)z=9+a \end{cases}$

Find a value of the parameter a, for which the system has the only one solution, and find this solution.

Problem 1.5 Given the system: $\begin{cases} x+2y+3z=5 \\ 2x+7y+(a+7)z=12 \\ x-y+\left(a^2-a-7\right)z=4a+15 \end{cases}$

Find such value of the parameter a, the system has an infinite number of solutions, and find the general solution for this value of a.

Problem 1.6 Given the system: $\begin{cases} 4x+y+4z=-4 \\ 8x+5y+(a+9)z=-5 \\ 4x-2y+\left(a^2-a-1\right)z=4a+1 \end{cases}$

Find the value of parameter a, for which the system has no solutions.

Problem 1.7 Write in a vector form the set of all the solutions of homogeneous system of equations:

$$\begin{cases} 2x - 2y + 3z = 0 \\ 3x + 4y - 3z = 0 \\ 7x + 3z = 0 \end{cases}$$

Problem 1.8 Write in a vector form the set of all the solutions of homogeneous system of equations:

$$\begin{cases} x - 3y + 2z = 0 \\ 5x - 15y + 10z = 0 \\ -4x + 12y - 8z = 0 \end{cases}$$

Problem 2.6 Given the system: $\begin{cases} k \cdot x + 5y + 5z = 3 \\ 10x + k \cdot y = 3 \\ 10x + k \cdot z = 3 \end{cases}$.

Find values of k, such that the system has only one solution and find it.

Problem 2.7 Given the system: $\begin{cases} k \cdot x + 4y + 4z = 2 \\ 8x + k \cdot y = 2 \\ 8x + k \cdot z = 2 \end{cases}$.

Find the values of k, such that the system has no solutions.

Problem 2.8 Given the system: $\begin{cases} k \cdot x - 3y - 3z = 2 \\ -6x + k \cdot y = 2 \\ -6x + k \cdot z = 2 \end{cases}$

Prove, that for $k = -6$ the system has infinite number of solutions, and find the general solution in this case.